

Abstract

Probability density functions (PDFs) can be understood as functional data carrying relative information. As such, standard methods of functional data analysis (FDA) are not appropriate for their statistical processing. They are typically designed in the L_2 space (with Lebesgue reference measure), thus cannot be directly applied to densities, as the metrics of L_2 does not honor their geometric properties. This has recently motivated the construction of the so-called Bayes Hilbert spaces, which result from the generalization of the Aitchison geometry for compositional data to the infinite dimensional setting. More precisely, if we focus on PDFs restricted to a bounded support (that is mostly used in practical applications), they can be represented with respect to the Lebesgue reference measure using the Bayes space of positive real functions with square-integrable logarithm. The reference measure can be easily changed through the well-known chain rule and interpreted as a weighting technique in Bayes spaces. Moreover, it impacts on the geometry of the Bayes spaces and results in so-called weighted Bayes spaces. The aim of this contribution is to show the effects of changing the reference measure from the Lebesgue measure to a general probability measure focusing on its practical implications for the Simplicial Functional Principal Component Analysis (SFPCA). A centered log-ratio transformation is proposed to map a weighted Bayes spaces into an unweighted L_2 space (i.e. with Lebesgue reference measure), thus it enables to apply standard statistical methods such as SFPCA on PDFs.