Abstract

For circular data, one frequently used continuous distribution is the von Mises distribution. This name refers to what is the $k = 1$ case of the Generalized von Mises distribution (GvM_k), where k connects to what types of modality can be incorporated in the model. Unimodality, bimodality, trimodality etc. are what we refer to as "types of modality". The pdf of the GvM_k includes a normalizing constant which cannot be expressed in closed form, but approximations are known for smaller values of k. For larger values of k, this is a limitation of the model.

We propose a lattice version of the GvM_k which here is used for count data, taking values of sectors of the circle. The finer the radial grid, the closer to the continuous GvM_k this distribution gets. For the lattice GvM_k, the normalizing constant is known explicitly, and any type of higher order modality can be included in the model. The pdf of the GvM_k typically includes types of modality in a sequential manner, meaning that if a trimodality component is included, components for unimodality and bimodality also are included. In the lattice model, we allow for including some, but not necessarily all, types of modality of a sequence. This approach results in a flexible model with no limitation on types of modality, allowing for symmetry as well as asymmetry of the modes, while still being in the von Mises framework.

In this setting, model selection becomes a problem of choosing an appropriate combination of types of modality to include in the model. We present one possible procedure of hypothesis testing for assessing what combination of types of modality should be included in the model.